Abstract: Well-designed and well-operated postharvest mechanical handling processes are vital if fresh fruits are to enter markets with minimal damage. The susceptibility of fruit to quality loss during handling is largely determined by mechanical contact and damage. This article reviews current understanding concerning the effect of impact and other mechanical contacts on fruits. It also discusses the major mechanical models that represent the interaction between fruits or a fruit and a surface. In total, 25 mechanical models associated with compression and impact behaviors of fresh fruits, mainly characterizing such behavior between 2 deformable spherical bodies and between a deformable spherical body and a rigid plate are reviewed. Finally, possible future research directions are discussed. The main challenge in fruit handling is to recognize and prevent microscopic damage to fruits, especially internal tissue/cell damage and microcracks resulting from multiple/repetitive impacts and compressions.

Keywords: compression, fruit, impact, mechanical damage, mechanical model, postharvest handling

Introduction

Half-ripe or nearly ripe fresh fruits at farms will be hand- or machine-harvested and go through a series of mechanical handling processes such as washing, grading, packaging, transportation, and finally, sale in markets. There are 3 main features during postharvest handling: (i) random compression and impact takes place incessantly among the fruits, between the fruits and from rigid container wall/subconveyor components during conveying and transportation, and during the free-fall drop of fruits on grading and in packaging equipment, possibly acting at the same location on the fruit; (ii) the storage conditions, with temperature and humidity fluctuating continuously; and (iii) fruits continue to ripen. Therefore, mechanical actions do have a large influence on physiological changes in the postharvest handling of fruits. In order to improve the textural quality of fresh fruits in markets, postharvest handling processes are performed at low temperatures that delay fruit ripening (Ahmad and others 2001; Luengwilai and Beckles 2013). Some packaging such as foam net sleeves and plastic trays are used to relieve compression and impact events (FAO 1995; Scetar and others 2010). Macroscopic mechanical damage of fruits is in this way greatly reduced. However, recognition and prevention of microscopic damage to fruits, especially internal tissue/cell damage and microcracks that result from repetitive compression and impact are still challenges to the food industry (Nicolai and others 2014; Opara and Pathare 2014).

Microscopic damage may lead to accelerated rot of a whole fruit, which is a food safety and economic issue (Li and Thomas 2014). At each stage of the handling process, flow damage may occur, the compound effect of each of these stages leading to a real and substantial loss of quality of the product. Many fruits with apparently little damage during harvesting are subsequently discarded and losses in the harvest-consumption system might be as high as 51% (FAO 2003). Reducing the amount of microscopic mechanical damage as a result of compression and impact during mechanical handling will further reduce the amount of fruit discarded (Li and Thomas 2016). Some scientific research and some practical experience shown that it will be possible to take action to mitigate such damage through an accurate understanding of how external compression and impact forces cause microdamage to fruits (Opara and Pathare 2014). The objective of this paper is to review the mechanical models associated with the compression and impact behaviors of fresh fruits during postharvest mechanical handling. Their uses in equipment design, textural evaluation, and damage prediction as well as future research directions are discussed.

Applications of Compression and Impact Mechanics in the Agro-Industry

Compression and impact contact mechanics of fruits is a key branch of biomaterial mechanics concerned with the behavior of biophysical bodies when subjected to contact forces or displacements. Fruit mechanics is usually characterized by the rigid plate-probe uniaxial compression test (Li and others 2011; Luca and others 2015; Fernando and others 2016), impact tests such as the free-fall drop test (Christophe and others 2014; Lien and Ting 2014), and the pendulum impact test (Van Zeebroeck and others 2007b). The major difference between compression and impact tests is that the rate of strain is instantaneously high during an impact, but can be controlled at a given level for a chosen time.
During compression (Salman and others 2007). In either case, the predominant assumption for the evaluation of fruit mechanical parameters is that small deformations of fruits during application of forces are elastic–plastic or viscoelastic–plastic. The factors affecting fruit mechanics mainly include temperature, humidity, ripeness stage, and cultivar (Van Zeebroeck and others 2007a). Some basic applications of knowledge and understanding of compression and impact mechanics of fruits are detailed in the following subsections.

**Design of mechanical handling equipment**

The physical properties of fruit, which are regarded as a part of contact mechanics in food engineering (Barbosa-Canovas and others 2006), are an important consideration in the design of postharvest mechanical handling facilities. The physical and mechanical properties of fruits are main determinants of intelligent harvesting strategies (Li and others 2011, 2013b), the size, material, power, and mechanical structure of grippers (Baeten and others 2008; Font and others 2014), and the design of separation devices and conveyor chains in fruit harvesting machines (Caprara and Pezzi 2011; Li and others 2012; Gonzalez-Montellano and others 2014). They are also necessary in the design of hoppers and sponge-drying sheets in washing machines (Ai-Katary and others 2010), the gap between adjacent rotary brushes in waxing machines (Zeng 2012), the structure and power of grading screws and separator devices, the material and structure of cushioning and shock absorbing devices (Brown and others 1990; Guter and others 1991), and the control mode and geometric size of conveyor devices in sorting and packaging lines (Pla and others 2001; Zhao and Li 2009). All these designs are intended to enhance efficiency and profitability.

**Prediction of mechanical damage**

Mechanical damage is only a qualitative term. From a material science viewpoint, fruit mechanical damage is the failure of a biomaterial or internal structure, and as such it is closely dependent on fruit mechanics. It is not surprising therefore that some impact and compression mechanical parameters, such as maximum acceleration, applied energy, absorbed energy, or peak force, which are obtained from free-fall drop (Yuwana and Duprat 1998; Yousefi and others 2016), pendulum impact (Van Zeebroeck and others 2007a; Ahmadi 2012), compression tests (Babaranis and Ige 2012), and electronic sensing devices (Siyami and others 2008; Chen and Yazdani 1991; Barreiro and others 1997), are usually used to evaluate quantitatively the degree of mechanical damage of fresh fruits. Another objective approach is to predict susceptibility to mechanical damage of fresh fruits by logistic regression modeling, to establish a functional relationship between the fruit handling conditions (such as mass, size, ripeness, cultivar, storage time, harvesting date, and loading position) and mechanical parameters (including peak force, impact/compression energy, absorbed energy, firmness, restitution coefficient, and stress relaxation) and the resulting damage evaluated as a binary response (Desmet and others 2002, 2003; Bieleza and others 2003; Blahovec and others 2004; Li 2013). The restitution coefficient refers to the ratio of the rebound velocity of the pendulum arm to its impact velocity (Van Linden and others 2006). Existing damage prevention models mainly result from statistical methods that are used to describe the effect of each potential damage mechanism. Most of the statistical models have been reviewed by Li and Thomas (2014). These statistical methods are observational studies and as such cannot determine causation and cannot be used to identify the key performance indicators (KPIs) for the agro-industries. However, mechanical models as discussed in this paper describe the fruit and its performance in a model with a random variable describing the mechanical parameters. By a careful design of an experimental study, it is possible to apply the law of statistical inference to deduce the causal link for mechanical damage from first principles. Such models enable the determination of KPIs for easy implementation in agro-industries.

In addition, impact and compression mechanical parameters are also used to develop finite element models for the simulation of internal mechanical damage of fresh fruits. The mechanical simulation of fruits with a multiscale geometrical model linking the macroscopic (whole fruit) scale through the mesoscopic (tissue) scale to the microscopic (cellular) scale (Fratzl 2003; Aizenberg and others 2005) would be scientifically and commercially valuable in understanding how external mechanical damage to fruit causes internal cellular damage and other changes that lead to bruising. There are some kinds of fruits such as apple (Dintwa and others 2008), watermelon (Sadrnia and others 2008), and tomato (Li and others 2013a, 2017) for which multtissue geometrical models have been developed to simulate the complex impact and compression mechanical behavior. More realistic fruit models would be more accurate in simulations using finite element analysis, but they would take a long time to compute, maybe even unachievable within the limits of existing computing power (Li and others 2015).

**Characterization of texture of fresh fruits**

Some impact and compression mechanical properties can be used to characterize quantitatively the generalized texture of fresh fruits (Abbott 2004; Ramallo and Mascheroni 2012; Li and others 2013c). Fruit firmness will decrease with increasing ripeness. Chen and De Baerdemaeker (1995) and Lien and others (2009) proposed that the ripeness and firmness of fruit can be assessed nondestructively using various impact mechanical parameters, including peak force, ratio of peak force to time-to-peak, coefficient of restitution, contact time, and frequency spectrum under free-fall drop and pendulum impact tests.

Texture profile analysis (TPA) is a popular method to characterize the texture of fruits by mechanical parameters extracted from the force–displacement data of a 2-cycle loading-unloading test (Bourne 2002; Rahman and Al-Farsi 2005; Tiago and others 2016). The main textural characteristics include hardness, fracturability, cohesiveness, springiness, chewiness, and resilience (Al-Haq and Sugiyama 2004; Rosenthal 2010). However, it is noticeable that the texture of a fruit is unique at a given time, while the values of mechanical parameters from a compression test depend on operational settings such as loading speed and probe diameter. In consideration of this problem, the mechanical parameters chosen to define textural characteristics should not be affected significantly by the operational settings and the size and shape of fruits. Trinh and Glasgow (2012) proposed: (i) the hardness and chewiness of food products may be defined as the Young’s modulus and rate of breakdown, respectively; (ii) the loading and unloading speed must be the same in order to compare energies during the loading and unloading phases of the cycle; and (iii) the preferred maximum distance of compression is best chosen between the elastic limit and the break point.

Above all, impact and compression tests provide an objective tool to gather insights into the relationship of fruit quality to mechanical properties (Blahovec 2001). Some horticulture scientists can accurately determine by impact and compression tests whether some newly cultured fruit cultivars are firm enough and suitable
The 2 terms (creep and stress relaxation) are sometimes used interchangeably, although they are really different. On the basis of the assumption that the stress relaxation can be treated as creep under decreasing stress, it has been proposed that the stress relaxation of a material can be predicted from known creep parameters and vice versa (Oman and Nagode 2014). Further investigations need to be performed to validate this interesting idea.

As the typical stress relaxation curve is fitted by a generalized Maxwell model, including a spring and 2 Maxwell units in parallel, and the typical creep curve is fitted by a generalized Kelvin model, including a spring, a dashpot, and a Kelvin unit in series, Figure 1a shows an implication mapping from the subterm in the stress–time equation to the segment in the stress–time curve, and Figure 1b shows an implication mapping from the subterm in the strain–time equation to the segment in the strain–time curve. It has been proven that the stress–relaxation behaviors of tomato cells can be imitated approximately by a 5-element generalized Maxwell model (Li and others 2016), and the creep properties of apple and tomato tissues imitated approximately by 4-element and 6-element generalized Kelvin model, respectively (Robert and David 1995; Chakespari and others 2010). Rheological models, including springs and dashpots in series and/or parallel, are mainly used to explain qualitatively the macroscopic viscoelastic behavior of fruit materials in an approximate range (Barrett and others 1998; Chen and others 2012; Diamante and Umemoto 2015). The springs represent the solid components of the fruit material and account for its elastic behavior under loading, while the dashpots represent fluid components and account for the viscous behavior of the fruit material under loading (Figueroa and others 2013). It is reported that these rheological models can also be used to explain the viscoelastic behavior of fruit materials at the molecular level. Alvarez and others (1998) proposed that the instantaneous elastic modulus of potato tissue could be associated with the internal pressure of cells, and the 2 Kelvin units in a 6-element Burgers model appeared to reflect the viscoelastic properties of pectin substances and hemicelluloses in the cell wall. Bucher and others (2008) proposed that the spring in a Maxwell model represents the deformation of peach tissue that occurs due to bending and stretching of interatomic bonds, and the immediately unrecoverable extension of the dashpot represents the result of intermolecular slippage.

Fruit materials will have a critical strain value under the action of external forces (Vincent 2004; Wang and others 2006). Below the critical strain, the deformation of fruit materials will be viscoelastic. When fruit material is subjected to a rapid loading force, the...
Mechanical models of compression and impact...

Dashpots do not respond quickly enough and the characteristics of the material are mainly to be identified with the response of the springs. Alternatively, when a fruit material is subjected to a slow or quasi-static loading force, the dashpots have enough time to respond, and the characteristics of the material are identified by the response of both the springs and dashpots. The slower the loading speed, the more significant the viscous response. Therefore, during an impact test, the fruit will mainly experience a "spring" elastic response that will not change with impact velocity over the duration of the impact. However, during a slow compression test, the "spring-dashpot" viscoelastic response is a function of time, and by controlling the velocity of the compression, it is possible to extract the maximum amount of information (Andrews and others 2013). Additionally, at higher temperatures, the viscosity in the dashpot decreases resulting in greater extensions, while at lower temperatures, the dashpot becomes more viscous and failure occurs before appreciable extension (Almagableh and others 2009).

Plastic behavior

Above the critical strain value, continuously increasing external forces will cause plastic deformation of the fruit materials; with further increasing external forces, failure such as cell burst (Blewett and others 2000), tissue/cuticle breakage (Bagrell and Neinhuis 2005; Li and others 2012), and/or fruit rupture will occur (Matas and others 2004; Li 2013). There has not been much investigation of the plastic/failure mechanics of fruit materials. Holt and Schoorl (1982) summarized that the failure in many fruits and vegetables can be classified as cleavage, slip, and bruising. Cleavage is a normal stress failure phenomenon, while slip and bruising are shear stress failure phenomena. In terms of the fracture mechanics of food, a key parameter, called the critical stress intensity factor $K_{IC}$, is described by Eq. (1) (Vincent 2004) where $a$ is the cleavage depth of indentation made by a tooth biting on the food item and $\sigma$ is the fracture strength. Vincent (2004) validated the hypothesis that the hardness of fruit and vegetables can be assessed by the critical stress intensity factor; so, it is possible to obtain the fracture strength of fruits by Eq. (1) and some related mechanical data from a compression test. The slip failure will take place on planes of maximum shear stress, namely, failure follows the Tresca criterion of Eq. (2), where $\tau_{\text{slip}}$ is the slip strength and $\sigma_1$ and $\sigma_3$ are the maximum and minimum principal stresses, respectively. The bruising failure is the bursting of fruit cells (Holt and Schoorl 1982), and the bruising strength is suggested by those authors as being described by Eq. (3), where $\tau_{\text{bruising}}$ is the bruising strength and $\sigma_2$ is the 2nd principal stress. Hence, Holt and Schoorl (1982) illustrated that horticultural materials do not fail according to 1 failure criterion alone, but they fail, on a rising load, according to the current strength boundary (the limit) first encountered by an expanding stress pattern. The conclusion is interesting, but it seems to exclude the von Mises failure criterion (Eq. (4)) where $\sigma_f$ is the yield strength, as described below:

$$K_{IC} = \sigma_b \sqrt{\pi a}$$

$$\tau_{\text{slip}} = (\sigma_1 - \sigma_3) / 2$$

$$\tau_{\text{bruising}} = \max ((\sigma_1 - \sigma_2) / 2, (\sigma_2 - \sigma_3) / 2)$$

$$\sigma_f = \sqrt{\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2}}$$

In the mechanics of materials, there are 2 classical criteria of failure: von Mises (maximum distortion energy) and Tresca (maximum shear stress) criteria. The von Mises failure criterion states the material yields when the maximum distortion energy in the fruit material equals the maximum distortion energy at yielding. The Tresca failure criterion implies that yield starts when the maximum shear stress in the fruit material equals the maximum shear stress at yielding (Gere 2004). According to the equations of von Mises failure criterion and Tresca failure criterion, Iheue and Mgbemena (2017) concluded that the yield strength of orange fruit is 0.03 and 0.01 MPa, respectively. Some researchers have stated that the maximum difference in yield strength between the von Mises and Tresca criteria is about 15%, and the von Mises criterion is more suitable for ductile materials (Gere 2004). Fruits and their tissues, cells, and cuticle can be regarded as ductile biomaterials because their Poisson's ratios are within the range of 0.2 to 0.5 (Kuna-Broniowska and others 2012). Therefore, the von Mises failure criterion is always used in finite element analysis to predict mechanical damage behavior of different types of fruits, tissues, or cells. Examples are sunflower fruits (Hernandez and Belles 2007), watermelon (Sadnina and others 2008), apple (Celik and others 2011), and tomato (Li and others 2013a).

With the improvement of instrument detection technologies and failure theories (Yu 2002; Opara and Pathare 2014), the failure characteristics of fruit materials in the uniaxial tension/compression tests, especially yield at the micro-/nanoscale, should be investigated further before choosing the most appropriate failure theory. Additionally, with increasing fruit ripeness, the chemical components and the contents of cells change (Prasanna and others 2007), the cells in tissues get bigger, and cell–cell adhesion in tissues becomes weaker (Wang and others 2006), so the failure characteristics at the microscale level at different ripeness stages of fruits may be different. This is also worth further investigation.

In summary, fruit materials are viscoelastic–plastic and their mechanical response depends on the loading speed of external forces, magnitude of deformation, ripeness, and storage temperature. Therefore, we recommend that in the literature concerned with compression and impact testing, the compression speed/initial impact velocity, experimental temperature and humidity, and fruit ripeness (USDA 1931 to 2008) should be listed fully in the Materials and Methods sections of papers. The values of force, displacement/fractional deformation and energy at the elastic limit, yield point, and rupture point, which can be directly extracted from force–displacement curves, are similarly essential in the Results section of every paper. If mechanical damage mechanisms of fruits are to be investigated more deeply, the viscoelastic–plastic mechanical properties of fruit biomaterials (such as apparent/instantaneous/equilibrium elastic moduli, shear modulus, and yield strength and strain) need to be predicted using some choice of contact model, as referenced below. Furthermore, the compression and impact to fruits may be multiple or repetitive, that is, a fruit may experience more than 1 impact/compression, or 2 or more impacts at the same location at the surface of the fruit. Therefore, some multiple/repetitive compression and impact models are also reviewed in the following sections.

Compression Mechanics

Compression always occurs during transportation because fruit at the bottom of a container will be subject to static compressive forces. Fruits at the top or bottom of a container on a truck/lorry might be subjected to additional static compressive forces from adjacent containers, if these are not well-designed. Compression
Fruits experience 3 stages during compression (Figure 2a and 2b): (i) OA—nondestructive elastic deformation, (ii) AB—deformation after biyoyielding, and (iii) BC—deformation after rupture. Force–deformation data can be accurately recorded in real time. Point A is defined as the elastic limit. Beyond that point, permanent tissue damage begins. There may be a bioyield point B where cells start to rupture or to move with respect to their neighbors before the peel, if any, is broken (Sirisomboon and others 2012), causing a noticeable decrease in slope. Point C marks rupture, where major tissue failure causes the force to decrease substantially (Abbott 2004). The explanation for stages AB and BC still need to be validated further by combining probe compression data, high-speed imaging, and other internal damage detection technologies, as reviewed by Opara and Pathare (2014). The shape, size, internal structure, and content of chemical components of fruits are always different because of differences in the growing environment, even for the same cultivar. This means that force–deformation curves of several fruits show obvious differences, as evidenced by the typical standard deviations on measurements of some mechanical parameters such as rupture force (Kilickan and Guner 2008; Altuntas and Ozturk 2013). The fruits containing a mixture of parenchyma and fiber in flesh tissues or stone cells and some locules have quite jagged force–deformation curves during compression tests. At present, if a fruit is compressed by a rigid plate probe, the resulting force–deformation curve cannot be converted into the true stress–strain relationship of the fruit material (Pallottino and others 2011). Therefore, it is extremely important to have mechanical models based on the compression of a spherical object by a plate to assist in the determination of the mechanical parameters (such as apparent elastic modulus and yield strength) of fruit materials.

**Compression models**

The compression of an approximately spherical fruit by another approximately spherical fruit can be simplified into compression between 2 deformable spherical bodies, and the compression from a rigid plate onto an approximately spherical fruit can be simplified into a compression between a rigid surface and a deformable spherical body. When 2 deformable spherical bodies are brought into contact and the compression force is increasing, the initial single contact point will expand into a finite area. Some related quasi-static compression models used to characterize the relationship between mechanical behavior and geometry of contact bodies are listed in Table 1 and 2.

![Figure 2–Compression test of whole fruit and a typical force–deformation curve. (A) A fruit is compressed by a rigid probe and (B) a typical force–deformation curve for compressed plum (Esehaghbeygi and others 2013). A: elastic limit point; B: biyoyield point; C: rupture or massive tissue failure point.](image)

**Table 1—Quasi-static compression models between 2 deformable spherical bodies**

<table>
<thead>
<tr>
<th>Number</th>
<th>Regime</th>
<th>Models</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Elastic–plastic (sphere– sphere contact)</td>
<td>$\delta = \frac{r^2}{R} = \left( \frac{9}{16} \frac{F^2}{R E^{2/R^2}} \right)^{1/3}$</td>
<td>Circular point contact $1/R = 1/R_1 + 1/R_2$ $1/E^* = (1 - \mu_1^2)/E_1 + (1 - \mu_2^2)/E_2$</td>
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<td></td>
<td>$1.6 \sigma_y = \frac{6F E^{2/R^2}}{\pi^3 R^2}^{1/3}, \tau_{max} = \sigma_y 2$</td>
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<tr>
<td>2</td>
<td>Elastic–plastic (ellipsoid– ellipsoid contact)</td>
<td>$\delta = \left( \frac{9}{16} \frac{F^2}{R E^{2/R^2}} \right)^{1/3} F_2(R'/R''$)</td>
<td>Elliptical point contact $R_e = (R/R'')^{0.5}$ $1/R' = 1/R_1' + 1/R_2'$ $1/R'' = 1/R_1'' + 1/R_2''$ $R'/R'' = (r_1/r_2)^{1.5}$</td>
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<td></td>
<td></td>
<td>$1.6 \sigma_y = \frac{6F E^{2/R^2}}{\pi^3 R_e^2}^{1/3} \left[F_2(R'/R'')\right]^{-2/3}$</td>
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Notes: Definitions of compression mechanical parameters in equations are shown in the Nomenclature section. Models 1 and 2: Johnson (1987).
Table 2—Quasi-static compression models of a sphere loaded by a rigid plate

<table>
<thead>
<tr>
<th>Number</th>
<th>Regime</th>
<th>Models</th>
<th>Note</th>
</tr>
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<tbody>
<tr>
<td>3</td>
<td>Elastic–plastic (sphere–rigid plate contact)</td>
<td>[ F = \sqrt{2R^3E_1^3} \left[ 1 - \frac{\mu^2}{1 - \mu^2} \right]^{1.5} ] [ 1.6\sigma_y = \left( \frac{6F_yE_1^2}{\pi^2R_1^2(1 - \mu^2)} \right)^{1/3} ]</td>
<td>Fractional deformation &lt; 10%</td>
</tr>
<tr>
<td>4</td>
<td>Elastic–plastic (sphere–rigid plate contact)</td>
<td>Equations for perfect slip contact (model 4a) [ \alpha_d = 0.38 + \mu_1/3, ] [ \delta_y = \left( \frac{\pi C_\infty}{2} \right)^{1/2}, R_1 F_y = \frac{\pi^3}{6} C_\infty \sigma_y \left( \frac{R_1\sigma_y}{E_1} \right)^2 ]</td>
<td>Ductile materials 0.2 &lt; \mu \leq 0.5, [ C_\infty = 1.2 + 1.3\mu_1 ]</td>
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<td></td>
<td>FEM for full stick contact (model 4b)</td>
<td>The sphere was divided into 3 different mesh density zones: Zone I &lt; 0.01R_1; Zone II &lt; 0.015R_1; Zone III &lt; 0.1R_1; Zone IV &lt; 0.15R_1; [ 200 &lt; \frac{E_1}{\sigma_y} &lt; 1000, 0.2 \leq \mu \leq 0.5. ] Failure criterion: von Mises.</td>
<td>Simulation software: ANSYS 8.0</td>
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<td></td>
<td>FEM for multiple loading-unloading (model 4c)</td>
<td>The sphere was divided into 4 different mesh density zones: Zone I &lt; 0.01R_1; Zone II &lt; 0.05R_1; Zone III &lt; 0.1R_1; Zone IV &lt; 0.15R_1; Zone V &lt; 0.2G_1. [ \mu_1 = 0.02G_1. ]</td>
<td>Simulation software: ANSYS 8.0</td>
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<td>5</td>
<td>Elastic (sphere–rigid plate contact)</td>
<td>[ \delta = \frac{3(1 - \mu^2)}{4E_1} \left[ 1 + \frac{B\sigma^2}{8R_1^2 (R_1 + U)} \right]^2 \frac{F}{\alpha} ] [ -f(\alpha')A = \frac{\pi E_1}{\alpha} \left[ 1 + \frac{B\sigma^2}{5R_1^2 (R_1 + U)} \right]^2 F ]</td>
<td>A = ( (1 - \gamma^2) / (1 - \gamma + \gamma^2 / 3) ) [ B = (1 - \gamma / 3) / (1 - \gamma + \gamma^2 / 3) ] [ \gamma = \frac{\lambda}{R} ] [ \alpha' = \sqrt{R^2 - (R - \frac{\delta}{\gamma})^2 + U} ] [ f(\alpha') = \frac{2(1 + \mu_1)R_1^2}{(\sigma^2 + 4R_1^2)^{1/5}} + \frac{1 - \mu_1^2}{(\sigma^2 + 4R_1^2)^{1/5}} ]</td>
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<tr>
<td>6</td>
<td>Viscoelastic (sphere–rigid plate contact)</td>
<td>[ F = \frac{2\sqrt{2R^3E_1}}{3(1 - \mu^2)} \delta_1^{1.5} \left[ G_\infty + \sum_{i=1}^n G_i e^{-t_0/t_r} \right] ] [ E_0 = 2 \left( G_\infty + \sum_{i=1}^n G_i \right) (1 + \mu_1), E_\infty = 2G_\infty (1 + \mu_1) ]</td>
<td>[ \varepsilon = \frac{\delta}{\sum R_1}, k_1 = \text{constant} ]</td>
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<td>7</td>
<td>Elastic (spherical shell–rigid plate contact)</td>
<td>[ \sigma_y = 6.3E_1^{3/4}, F = \frac{2\pi d E_\infty R_1}{3G_0} ]</td>
<td>[ E_\infty = \text{Elastic modulus of cell wall} ] [ d = \text{Initial wall thickness} ] [ t_0 = \text{Initial cell radius} ] [ \bar{\lambda}_0 = \text{Initial stretch ratio} ]</td>
</tr>
<tr>
<td>8</td>
<td>Elastic–plastic (spherical shell–rigid plate contact)</td>
<td>Dimensionless analysis [ \text{Input: } \tilde{F} = \frac{F}{E_0 d_0 \lambda_0^2}, \tilde{\varepsilon} = 0.538 / \tilde{F}_{\lambda_0} ] [ \text{Output: } \tilde{F} = \tilde{\varepsilon} \text{ curve} ]</td>
<td>[ E_\infty = \text{Elastic modulus of wall} ] [ d_0 = \text{Initial diameter of sphere} ] [ \tilde{\lambda}_0 = \text{Initial stretch ratio} ]</td>
</tr>
<tr>
<td>9</td>
<td>Elastic–plastic (Sphere/spherical shell–rigid plate contact)</td>
<td>FEM for cell compression (model 9a) [ \text{Input: } (i) \text{ basic model definition, (ii) contact model, (iii) material properties of cell wall and protoplasm, and (iv) constraints and loads.} ] [ \text{Output: } (i) \text{ force-fractional deformation curve during compression and (ii) deformed shapes of cell at different compression stages.} ]</td>
<td>Simulation software: COMSOL Multiphysics 3.2</td>
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<td>DEM for tissue compression (model 9b)</td>
<td>DEMeter- in-house code</td>
<td>Simulation software: COMSOL Multiphysics 3.2</td>
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<td></td>
<td>FEM for fruit compression (model 9c)</td>
<td>Simulation software: ANSYS 10.0 and ABAQUAS 14.5</td>
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</tbody>
</table>

Notes: Definitions of compression mechanical parameters in equations are shown in the Nomenclature section. Model 3: Johnson (1987); Model 4: Brizmer and others (2006); Model 5: Tatara (1993); Model 6: Lee and Radak (1960); Model 7: Blahovec (1994); Model 8: Wang and others (2004); Model 9: Dintra and others (2011); U and others (2013a, 2017); Van Liedekerke and others (2011); FEM—finite element method; DEM—discrete element method; SPH—smoothed particle hydrodynamics; 2D—two-dimensional.
Compression between 2 deformable spherical bodies. Hertz (1882) proposed some classic equations (model 1) to characterize the quasi-static contact mechanical behavior of 2 elastic–plastic spheres during compression, namely, Hertz contact theory. The assumptions are: (i) the contact area is circular and the size of the contact area is small compared with the radius of the spheres; (ii) both contacting surfaces are smooth and frictionless; and (iii) the deformation is elastic up to the elastic limit (Figure 2b). If the 2 contact bodies have different radii, the contact area will be elliptical and the extended model 2 (Table 1) will be more suitable for characterizing the contact behavior. The additional terms \( F_1 (R'/R) \) and \( F_2 (R'/R') \) in model 2 imply this and are regarded as “correction factors” to allow for the eccentricity of an ellipse (Johnson 1987). To a first approximation, they may be taken as 1. The stress distribution on the contact area is regarded as parabolic with a maximum value at the center. The stress falls to zero outside the area of contact (Johnson 1987). When the yield stress, \( \sigma_y \), is exceeded for an elastic–plastic sphere, any further deformation will be plastic and nonrecoverable. This provides an important mechanism for loss of energy and permanent damage to the fruit.

The force at which the yield stress is equal to the maximum stress in the fruit is obtained at a force, \( F_y \). The equations for yield strength of spherical bodies are also listed in Table 1, based on the Tresca yield criterion. By experiments and numerical simulations, Yang and Chun (1969) confirmed that the quasi-static Hertz contact theory can be extended to analyze the contact of elastic and viscoelastic bodies, viscoelastic bodies, and elastic and plastic bodies. These 2 models can be used to predict the contact mechanical behavior of 2 approximately spherical fruits during compression. However, it is difficult to measure and validate the contact force and deformation of 2 fruits during compression by existing technologies, so the models are not always used directly in the research of quasi-static mechanics of fruits.

Compression from a rigid plate onto a deformable spherical body. Model 1 can be simplified into model 3 (Table 2) when one of the elastic–plastic spheres is replaced by a rigid flat-ended probe (\( 1/R_2 \approx 0, 1/E_2 \approx 0 \)). The quasi-static mechanical properties of approximately spherical fruits, single cells, or particles compressed by a rigid plate probe are always evaluated by model 3 with the assumption of a circular contact area. Using this approach, Sirisomboon and others (2012) found the apparent elastic modulus of Momotaro tomato fruits at mature green, pink, and red ripening stages to be 5.6, 4.2, and 3.8 kPa, respectively. Shirvani and others (2014) showed that the apparent elastic modulus of 3 apple cultivars ranged from 2.2 to 3.4 MPa. However, some researchers have shown, with a bottom camera, that the contact area of a fruit and flat-ended probe is not circular but an ellipse (Pallottino and others 2011). In order to confirm whether model 3 is suitable for predicting the compression mechanics, it is necessary to investigate the effect of the shape of the contact area on the predictions of the contact models. For orange fruits, Huzeve and Mgbeumen (2017) assumed the contact area to be elliptical under axial and radial compression and then they evaluated the maximum contact pressure and the size of the contact area by a simplified model deduced from model 2. Perhaps, it would be more suitable to assess the mechanics of fruits compressed by a flat-end rigid probe using a new extension model developed from model 2. The equations for yield strength in models 1, 2, and 3 are based on the Tresca criteria.

Tatara and others (1991) concluded that model 3 applied for fractional deformations of <10% after an examination of the compression of rubber elastic spheres over a large range of displacements. However, the relationship between the fractional deformation of a sphere and the nominal strains in the material of which the sphere is made is unknown. Therefore, it is still difficult to give an exact critical nominal strain value for the application of model 3. By finite element analysis, Kogut and Etsion (2002) demonstrated that the evolution of the elastic–plastic contact between a sphere and a rigid flat surface can be divided into 3 stages based on the dimensionless ratio of the increasing deformation of a sphere under external forces to the elastic limit \( \delta / \delta_c \). In stage I \( \delta / \delta_c < \delta_c \), the plastic region develops a subsurface and the contact area is elastic. In stage II \( \delta_c < \delta / \delta_c < \delta_h \), the contact area is elastic–plastic. In stage III \( \delta / \delta_c > \delta_h \), the contact area is fully plastic. Models 1 to 3 assume that there is perfect slip at the contact when a deformable sphere is compressed by a rigid flat surface or another sphere, namely, there are no tangential stresses in the contact area. It is essential to extend this interesting conclusion to an analysis of the 3-dimensional elastic–plastic contact mechanics of fruits or single cells. Objectively, the real pressure distribution in the contact area between fruits and a probe surface can be measured by the tactile sensing system proposed by Herold and others (2001), so it is now possible to investigate the critical failure characteristics in the contact area of fruits during compression.

On the basis of model 3 and the von Mises failure criterion, the equations of model 4a (Table 2) were deduced by Brizmer and others (2006). These characterize the dimensionless depth of inception of yielding, critical displacement, and the load at inception of yielding of a ductile sphere compressed by a rigid flat surface under perfect slip condition. Then, a finite element model 4b was developed to solve numerically for the inception of yielding of a ductile sphere with different mechanical properties under full stick contact conditions. It was found that the radius of the contact area was insensitive to the choice of full-stick or perfect-slip contact conditions, and that the inception of yielding of ductile materials occurs along the line of symmetry where the stress is at its maximum (Brizmer and others 2006). When Poisson’s ratio is small, then the values of the depth, critical displacement, and load at yielding inception are lower for stick when compared to slip. At higher Poisson’s ratios, inceptions of yielding are insensitive to the stick/slip condition (Brizmer and others 2006). In investigations of damage mechanics of fruits, it is therefore important to obtain accurate Poisson’s ratios of fruits and fruit components and the actual contact type between fruit and probe. Kadin and others (2006) proposed a finite element model for multiple repeated loading-unloading of an elastic–plastic sphere compressed by a rigid flat surface (model 4c), and subsequent simulations covered a wide range of loading conditions far beyond the elastic limit. The results demonstrate that the majority of plastic deformation occurs during the first loading and that the secondary plastic flow may evolve during the first unloading. Since plastic contact is history-dependent, the occurrence of the secondary plastic flow depends strongly on the level of first loading, and furthermore, it is affected by Poisson’s ratio and Young’s modulus. The region of the secondary plastic flow may propagate during the first loading/unloading cycles; a steady state is reached after which the subsequent loading/unloading cycles become fully elastic. This conclusion is helpful in investigations of the mechanical damage of fresh fruits during multiple repeated compression and/or impact actions.

Hertz contact theory in model 3 applies for small percentage deformations of <10% (Yan and others 2009), and it cannot explain some large deformation phenomena. Hence, Tatara (1993) proposed model 5 (Table 2) to describe the large deformation of a
rubber sphere under diametral compression, considering nonlinear elasticity and lateral extension of the compressed sphere. Shima and others (1993) verified that model 5 can characterize deformations of 20% to 83% for compressions of a rubber sphere with the assumption of constant volume but cannot be used for small deformations. Yan and others (2009) discussed that model 5 overestimated the experimental force due to compression of agarose microparticles with increasing deformations up to 60%. The reason is that agarose microparticles may fail during microcompression at large deformations and cannot be regarded as rubber-like and purely elastic spheres. Fruit materials are viscoelastic-plastic, not purely elastic. According to model 5, it should be possible to investigate the failure of approximately spherical fruits or cells under large deformations in compression by comparing the force-deformation curves of fruits and rubber spheres.

For compression-holding tests, the contact region between a viscoelastic body and plate probe varies with time in the relaxation phase. On the basis of earlier Hertz–Maxwell models (Lee and Radok 1960; Mattice and others 2006), Yan and others (2009) proposed extension model 6 (Table 2) to investigate the viscoelastic properties of agarose microparticle spheres in compression-holding tests, and they concluded that the equilibrium elastic modulus in relaxation approximated the apparent elastic modulus in compression. The compression at small deformations at very low speed can be regarded as an inverse motion of stress relaxation. Based on this model, Li and others (2016) calculated the instantaneous and equilibrium elastic modulus, and the 1st and 2nd relaxation times of single tomato cells that are $0.65 \pm 0.28$ MPa, $0.22 \pm 0.08$ MPa, $0.48 \pm 0.05$ s, and $0.033 \pm 0.004$ s, respectively. Kim and others (2008) concluded that the instantaneous and equilibrium elastic modulus, and the 1st and 2nd relaxation times of apple flesh are $1.73 \pm 0.57$ MPa, $1.0 \pm 0.33$ MPa, $186 \pm 77$ s, and $17 \pm 7.5$ s, respectively. It has been shown that model 6 can be used to characterize the spherical indentation load-relaxation behavior of soft biological tissues with percentage deformations of about 7% to 15% (Mattice and others 2006). This model applies to linear viscoelastic materials with a time invariant Poisson’s ratio (Yan and others 2009).

For most round solid agricultural products, Blahovec (1994) proposed that there is a simple power relation between failure strength and elastic modulus, which is listed as model 7 in Table 2. If the fruit can be regarded as a closed isotropic elastic membrane (for example, skin) filled by fluid, the 2nd equation in model 7 can be used to describe approximately the mechanical behavior of a fruit compressed by a rigid plate. The elastic moduli of fruit skin predicted from the 2nd equation in model 7 are in very good agreement with values that have been obtained by direct tension of fruit skin: a secant modulus about 10 MPa for apple and 30 to 80 MPa for tomatoes (Blahovec 1994). Blahovec and Posva (1996) proposed the last 2 equations in model 7 for describing the role of internal stress in the compression of a berry-like cell between 2 plates. The cell model is made up of a thin elastic membrane and a simple liquid with 3 filling levels; it is considered that the turgor pressures in the overfilled, fully-filled, and underfilled cell models are $>0$, 0, and 0, respectively. The results showed that the filling levels/degrees of the cell have an obvious effect on its mechanics at low compression levels. In consideration of the decreasing change of turgor of fruit during storage, model 7 can be used to investigate the reasons for textural change of postharvest fruits and vegetables.

Wang and others (2004) proposed a dimensionless analysis method listed as model 8 in Table 2 to describe the elastic modulus of the cell wall and the initial inflation of single tomato cells. The cell is treated as a liquid-filled sphere with thin compressible, linear-elastic walls. Numerical simulations have led to predictions of force–fractional deformation data using the initial stretch ratio and elastic modulus as adjustable parameters. The initial stretch ratio is the ratio of the radius of the cell after inflation to the initial radius of the cell before inflation, that is, at zero turgor. The elastic modulus of cell wall can be determined if the simulated data from model 8 are fitted to experimental force–fractional deformation data in cell compression. It was found that the elastic modulus of the cell wall of single suspension-cultured tomato cells was in the range of 1.4 to 3.4 GPa (Wang and others 2004) and the elastic modulus of cell walls of tomato mesocarp was 30 to 80 MPa (Wang and others 2006). In dimensional analysis, the definitions of dimensionless mechanical parameters such as force, pressure, fractional deformation, and yield stress are not unique, and nonunique solutions would be possible for the elastic modulus (Smith and others 1998).

Dintwa and others (2011) modeled a tomato cell under compression as a liquid-filled sphere with a thin semipermeable and linearly elastic wall (model 9a in Table 2). The model was sensitive to wall thickness and internal pressure of the cell, but it was capable of accurately predicting force–deformation data of a tomato cell under compression, and also of its deformed shape. Van Liedekerke and others (2011) developed a model for a small piece of tomato tissue consisting of some adhering cells to investigate the mechanism of bruising in soft cellular tissue under excessive stress (model 9b in Table 2). The parenchyma cells in the tissue model include solid cell wall and liquid-like protoplast. The cell fluid is modeled by a particle hydrodynamics technique and cell adhesion is modeled by a nonlinear discrete element method. The simulation results showed that a low-viscosity protoplast will merely cause local damage of cells, while a highly viscous protoplasm will cause more structural damage of the tissue. The reason is that an increasing viscosity of protoplasm induces a cell to react as an overdamped system and the forces applied to the tissue are transmitted more in the loading direction and thus deeper into the tissue. Li and others (2013a, 2017) developed 2 multiscale finite element FE models to predict the internal damage of tomato fruits subjected to external compressive forces (model 9c). The 1st FE model was validated as being able to predict the evolution of internal mechanical damage regions of tomatoes with the increasing of external forces (Li and others 2013a). The 2nd FE model was validated as being able to predict the internal mechanical damage volumes of tomatoes under different external forces (Li and others 2017). It appears that finite/discrete element simulations are effective in predicting the mechanical damage behavior of spherical fruits, tissues, or single cells compressed by a rigid plate such as a flat-end probe. Furthermore, this method can be extended to investigate the internal mechanical behavior of other fruits, tissues, or cells besides tomatoes.

**Impact Mechanics**

In the postharvest handling of fresh fruits, multiple impacts usually occur in free-fall drop due to positional changes following different handling sequences within a packaging factory. Impacts always occur in collisions between fruits and fruits/handling equipment/containers during mechanical handling. These impact motions can be summarized into 2 groups: (i) collision of a fruit onto another and (ii) free drops of a fruit onto a rigid plate. The basic impact processes and related mechanical models are described in detail below.
Collision between 2 deformable spherical bodies

Basic collision processes. In considering the transport of fruits during postharvest handling, horizontal collinear collisions were selected for analysis. In general, 2 phases can be identified: compression I and restitution II, as shown in Figure 3. The 1st phase begins when the 2 fruits come in contact at state 1, and ends when the maximum deformation is reached at state 2, where the relative normal velocity is 0. The 2nd phase begins at state 2 and ends at state 3 when the fruits separate. Not every deformation is recoverable in the 2nd phase, because some are permanent and there is resulting energy loss (Gilardi and Sharf 2002). With respect to phase II, collision can be classified into: (i) perfectly elastic without energy loss; (ii) partially elastic, with energy loss and permanent deformation; and (iii) perfectly plastic, where all energy is lost and the deformation is permanent. The velocities of 2 fruits after collision can be calculated by the principle of conservation of linear momentum. The coefficient of restitution $e$ is defined as $-\left(\frac{v_1' - v_2'}{v_1 - v_2}\right)$ to measure the elasticity of a collision between 2 bodies, where $v_1$ and $v_2$ are the velocities of 2 fruits before collision and $v_1'$ and $v_2'$ are the velocities of 2 fruits after collision. For plastic collision, $e = 0$; for perfectly elastic collision, $e = 1$. During partially elastic collision (0 < $e$ < 1), a part of the initial kinetic energy is transformed into final kinetic energy and the residual part is energy lost, expended in wave propagation, plasticity, material damping, and other processes (sound, heat) (Stronge 2004). The coefficient of restitution depends on many elements, such as the geometry of the fruits in contact, approach velocity, material properties, collision duration, and possibly, friction (Gilardi and Sharf 2002; Minamoto and Kawamura 2009).

If the mechanics of collision of a fruit onto another are investigated, the values of coefficient of restitution, peak force, displacement/fraction deformation, and energy at elastic limit point, collision duration always needs to be determined. If the impact damage mechanism needs to be investigated further, the dynamic elastic–plastic properties of fruit materials such as apparent elastic modulus and yield strength need to be predicted using the following models. Investigations of the failure mechanics of collisions between spherical fruits are scarce. Yang and Chun (1969) confirmed that the relationship between loading force and deformation during compression, described by quasi-static Hertz contact theory, can also be used to explain the relationship between force and deformation during impact. Therefore, the majority of collision mechanics models have been derived from the quasi-static Hertz contact theory.

Impact models of horizontal collision. The horizontal collision of a fruit onto another is an example of impacts of deformable bodies. Some related impact models, which are listed in Table 3, illustrate the basic relationships between impact characteristic parameters (such as collision duration and velocity) and the mechanical and geometrical parameters of 2 elastic–plastic/viscoelastic spheres. In model 10, Johnson (1987) characterized the quasi-static impact behavior between 2 frictionless elastic spherical bodies by 3 equations. In model 11, it is assumed that the softer body between 2 elastic–plastic collision spheres will reach the limit of elastic behavior when the maximum contact pressure at the instant of maximum compression reaches the value $1.6\sigma_y$. The 2 equations show that the yield strength of 2 colliding bodies does not only depend on its material properties (such as mass, shape, and elastic modulus), but also depends on the relative velocity of the 2 bodies at impact, and the coefficient of restitution depends upon the severity of the impact (Johnson 1987).

In model 12 (Table 3), Thornton (1997) presented an analytical solution for the coefficient of restitution based on a theoretical equation for the normal contact interaction between 2 elastic–perfectly plastic spheres. The Hertz pressure distribution was assumed with a cutoff corresponding to the contact yield strength when the plastic deformation of the spheres occurs. The model is sensitive to the value input for the cutoff yield pressure. The failure assumption is different between models 11 and 12. According to model 12, Minamoto and Kawamura (2011) found that the deformation of spheres was underestimated in moderately high-speed impacts but overestimated in low-speed impacts. By finite element analysis, Wu and others (2003) showed that model 12 is suitable for the impact of small plastic deformation of spheres, while the restitution coefficient can be approximated to be proportional to $[(V_1/V_2)/\left(E\alpha/\sigma_y\right)]^{-0.5}$ for impacts of finite-plastic deformation.

Models 13 to 15 (Table 3) characterize the impact behavior of 2 viscoelastic bodies and the instantaneous impact force was described by several high-order polynomials, which were made up of a linear elastic term and a viscous damping term. Hunt and Crossley (1975) proposed that the damping energy loss is proportional to the cube of relative velocity of 2 viscoelastic bodies before collision (model 13). It is not certain how the effective damping coefficient $\alpha$ can be derived for the given equation, even knowing the viscous coefficients of individual bodies measured.
Table 3—Impact models of soft-spheres

<table>
<thead>
<tr>
<th>Number</th>
<th>Regime</th>
<th>Models Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>Elastic</td>
<td>$F = \frac{4}{3} R^{0.5} E^{1.5}$</td>
</tr>
<tr>
<td></td>
<td>$\varepsilon = 1$</td>
<td>$\gamma = \frac{15 m V_z^2}{16 R^{0.5} E^{0.4}}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$T = 2.948/V_z$</td>
</tr>
<tr>
<td>11</td>
<td>Elastic-plastic</td>
<td>$\frac{1}{2} m V_f^2 = 53 R^3 \sigma_{y}^5 / E^{4.4}$</td>
</tr>
<tr>
<td></td>
<td>$0 &lt; \varepsilon &lt; 1$</td>
<td>$\varepsilon = 3.8(\sigma_{y}/E)^{1.4}/(\frac{1}{2} m V_f^2 / \sigma_{y} R^3)^{1.6}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\alpha = 0.1 \text{ m/s}$</td>
</tr>
<tr>
<td>12</td>
<td>Elastic-plastic</td>
<td>$F = \frac{4}{3} R^{0.5} E^{1.5}$</td>
</tr>
<tr>
<td></td>
<td>$0 &lt; \varepsilon &lt; 1$</td>
<td>$V_f = 3.19 \left( \frac{\sigma_{y}^5 R}{E^{4.4} m^2} \right)^{0.5}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Plastic loading: $F = F_y + \pi \sigma_{y} R (\delta - \delta_y)$</td>
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<tr>
<td></td>
<td></td>
<td>$F_y = \frac{2}{3} \pi \sigma_{y}^2$</td>
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<td></td>
<td></td>
<td>$\varepsilon = 1.4 \left[ 1 - \frac{1}{6} \left( \frac{V_f}{V_z} \right)^2 \right]^{1/2}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$V_f/V_z + 0.9 [6 - (V_f/V_z)^2]^{1/2}$</td>
</tr>
<tr>
<td>13</td>
<td>Viscoelastic</td>
<td>$F = \frac{4}{3} R^{0.5} E^{1.5} \delta (1 + 1.5 \alpha \delta)$</td>
</tr>
<tr>
<td></td>
<td>$0 &lt; \varepsilon &lt; 1$</td>
<td>$\varepsilon = 1 - \alpha V_z$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\Delta E = \alpha m V_z^2$</td>
</tr>
<tr>
<td>14</td>
<td>Viscoelastic</td>
<td>$F = \frac{4}{3} R^{0.5} E^{1.5} \delta + 2 \kappa R^{0.5} \delta \varepsilon \delta$</td>
</tr>
<tr>
<td></td>
<td>$0 &lt; \varepsilon &lt; 1$</td>
<td>$\varepsilon = 1 - 1.94 \kappa R^{0.5} E^{0.6} (V_k/m^2)^{0.2}$</td>
</tr>
<tr>
<td>15</td>
<td>Viscoelastic</td>
<td>$F = \rho V_0^{1.5} + 1.5 A \rho \delta \varepsilon \delta$</td>
</tr>
<tr>
<td></td>
<td>$0 &lt; \varepsilon &lt; 1$</td>
<td>$V_0^{*} \approx 0.06 A \varepsilon^{-1} \rho^{-2} m^2$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\varepsilon = 1 - \gamma_1 V_0^{2.0} + \gamma_2 V_0^{4.0} \ldots$</td>
</tr>
<tr>
<td>16</td>
<td>Viscoelastic-plastic</td>
<td>Input: number, radius, density, color, initial position, and velocity of fruits;</td>
</tr>
<tr>
<td></td>
<td>$0 &lt; \varepsilon &lt; 1$</td>
<td>dimensions of box; parameters of contact force model Output: position and velocity of fruits; contact force of all fruits in time step. Further calculation:</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Damage depth = 5.67 x In (Peak force) – 18.99</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Damage volume = 15.81 x Peak force – 608.9</td>
</tr>
<tr>
<td>17</td>
<td>Viscoelastic</td>
<td>Input: 3D geometric model of multituessises, constitutive law (2 element</td>
</tr>
<tr>
<td></td>
<td>$0 &lt; \varepsilon &lt; 1$</td>
<td>generalized Maxwell model) and material properties (Poisson’s ratio, density, instantaneous elastic modulus, shear modulus and relaxation time, bulk modulus, and time constant). Output: Deformation time, rate of deformation time data of 2 apples during collision, rebound velocity of collision of elastic, or viscoelastic bodies, Further calculation: viscous coefficients of model 5, restitution coefficient of elastic and viscoelastic body.</td>
</tr>
</tbody>
</table>


by stress relaxation test. Kuwabara and Kono (1987) extended the theory of Hertz to materials having internal friction, namely, to a viscoelastic material by assuming: (i) the energy dissipation during impact is due to the viscosity in a sphere without plastic deformation; (ii) the total force acting on the contact area is made up of elastic force and frictional force; and (iii) the viscosity coefficient of sphere material is associated with volume deformation and shear (model 14). Under the same assumption of energy loss, Ramirez and others (1999) applied dimensional analysis to inelastic collisions of spherical particles (model 15) to develop an explicit expression for the coefficients of the series expansion of the restitution coefficient in terms of the impact velocity.

From a quantitative perspective, Stevens and Hrenya (2005) showed that the measured collision durations are within 10% of those predicted by model 10 for perfectly elastic spheres; the restitution coefficient and collision duration predicted by model 14 is in excellent agreement with experimental data; for plastic deformation, model 12 provided very good estimates for collision duration, but it overpredicted the dependency of the coefficient of restitution on the relative velocity just prior to collision of 2 elastic–perfectly plastic spheres. Impact tests between a spherical impactor (Van Zeebroeck and others 2003; Abedi and Ahmadi 2013; Opara and Pathare 2014) and fruit can be described by models 10 to 15, so that the impact failure behavior of fruits and
the external factors that affect mechanical damage of fruits can be investigated further by pendulum-based impact experiments. Using these models 10 to 15, it is possible to make some comparisons between model predictions and experimental data provided by pendulum-based impact tests, at least for spherical fruits, but research on this issue is scarce. It was usually assumed that no tissue damage had occurred when the force-time curve of a repeated impact, with the same impact energy as that of the 1st impact, was the same. The critical impact energy for potatoes was 0.024 J, for apples 0.010 J, and for tomatoes 0.006 J (Van Zeebroeck and others 2003). Fruits with higher stiffness, namely, lower ripeness, will have collisions of shorter duration, which reduces the risk of developing bruises (Van Linden and others 2006).

In model 16 (Table 3), Van Zeebroeck and others (2006a, 2006b, 2008) developed discrete element models to predict the impact damage of apples in boxes or bulk bins during transport and handling. Model 14 was defined as a contact force model and the physical parameters (initial position and velocity of apples) were input parameters for discrete element modeling (DEM) simulations. Bruise depth is the only output damage parameter that can be calculated unambiguously from DEM simulations because it is only defined by the magnitude of the acceleration pattern (Van Zeebroeck and others 2006a, b). Multiple impacts around the same location alter both bruise depth and diameter. However, the bruise surface area and volume were not predicted, because so far the impact positions data on the apple surface cannot be saved by a coordinate system. Similarly, Van Zeebroeck and others (2008) simulated the effect of truck load, bulk bin position, suspension type, and driving speed on the impact damage volume of apples by DEM. It was found that these parameters have important effects on the amount of apple bruising, and that driving speeds of <20 km/h do not lead to commercially significant bruising.

A major source of DEM errors (model 16) is inadequate accounting during collisions for all the energy of the individual objects in the system being modeled. In this regard, Dintwa and others (2008) developed some finite element models (model 17, Table 3) to conduct a detailed study of the collisions of apples. During modeling, the apple was regarded as a viscoelastic multi-tissue system and their constitutive law was a 2-element generalized Maxwell model. After simulation, it was shown that the absorption of dynamic waves excited during collisions can lead to substantial kinetic energy losses for soft and relatively large fruits, so any quasi-static models may lead to large errors. This study is close to the typical practical collision regimes of fruits during postharvest mechanical operations. Whether DEM or finite element method (FEM) simulations, the calculation method of viscous coefficients in the used force contact model is different from the original definition by Kuwabara and Kono (1987). Perhaps, this is another reason that all viscoelastic simulations still feature large discrepancies between experiments and simulations.

Drop of a deformable spherical body onto a rigid plate

Basic free-fall drop processes. Free-fall drops of fruits have some features similar to horizontal collisions but are not quite the same. As shown in Figure 4, there are 3 phases in a free-drop cycle. Phase I is the fruit-free dropping from a height \( h_1 \) to immediately contact the rigid surface prior to impact with a unique posture. With the action of gravitational acceleration \( g \), the instantaneous velocity of a fruit before impact reaches the maximum \( v_1 = (2gh_1)0.5 \) based on the law of conservation of energy. Phase II is the impact between the fruit and surface. The impact acceleration \( a \) is not constant and the velocity of the fruit changes from downward \( v_1 \) to upward \( v_2 = (2gh_2)0.5 \) immediately after impact. Meanwhile, the kinetic energy \( \frac{1}{2}mv_1^2 \) of fruit immediately prior to impact is fully transformed into the deformation energy \( \Delta E \). If the drop height \( h_2 \) is lower than the critical height for damage, then there will only be elastic deformation energy \( \Delta E_e \). Otherwise, the deformation energy will be made up of elastic deformation energy \( \Delta E_e \) and plastic deformation energy \( \Delta E_p \) (Horsfield and others 1972). The maximum deformation during impact can be measured by a high-speed camera (Stropek and Golacki 2015).

![Figure 4–Free-fall drop processes of a fruit. I, II, III, and IV are the 1st, 2nd, 3rd, and 4th stages in the free-fall drop test, respectively; \( h_1 \)—initial drop height; \( h_2 \)—rebound height; \( v_1 \)—downward velocity of fruit immediately prior to impact; \( v_2 \)—upward velocity immediately after impact; \( g \)—gravity acceleration; \( a \)—acceleration during impact; \( \Delta E \), \( \Delta E_e \), and \( \Delta E_p \)—deformation energy, elastic deformation energy, and plastic deformation energy, respectively; \( a_{max} \)—maximum acceleration; \( F_{max} \)—peak impact force; \( T \)—impact duration.](image-url)
Table 4—Impact models of free fall drop

<table>
<thead>
<tr>
<th>Number</th>
<th>Regime</th>
<th>Models</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>Elastic $e = 1$</td>
<td>$v_1 = - \sqrt{2gh_1}$</td>
<td>Assumption: $v_1 = v_1'$ (perfect rebound)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$F_{\text{max}} = m_1 \alpha_{\text{max}}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\alpha_{\text{max}} = 2 \times a_{\text{avg}} = 2 \times \sqrt{2gh_1}$</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>Elastic $e = 1$</td>
<td>$F(t) = -m_1 \pi \frac{\pi t}{2} \delta(t)$</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>$\delta(t) \approx \delta \sin \left( \frac{\pi t}{T} \right)$</td>
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<tr>
<td></td>
<td></td>
<td>$T = 2.94(t/v_1)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$a(t) = \frac{\pi}{2} \sin \left( \frac{\pi t}{T} \right) \frac{v_2 - v_1}{2}$</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>Elastic–plastic $0 &lt; e &lt; 1$</td>
<td>$\sigma_0^e = \frac{E_1^e V_f^e}{26(1 - \mu_1)}$</td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>Elastic–plastic $0 &lt; e &lt; 1$</td>
<td>$\tau_{y1} = 0.244(mgh_1)^{0.2} E_1^e 0.8 \rho^{-0.6}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\tau_{y2} = 0.244(mgh_1)^{0.2} \left( \frac{v_1}{2R} \right)^{0.8} K$</td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>Elastic–plastic $0 &lt; e &lt; 1$</td>
<td>Input: 3D geometric model of fruits, constitutive law (Hertz elastic contact model), material properties (Poisson's ratio, density, instantaneous elastic modulus, and/or shear modulus) and/or critical drop damage height. Output: (i) internal stress distribution and deformation situation of fruit during drop impact; (ii) contact force–time and deformation–time data during impact and rebound; and (iii) $F_{\text{max}}$ and $\alpha_{\text{max}}$ at the center of the fruit.</td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>Plastic $e = 0$</td>
<td>$F = \sqrt{\frac{E_1^e S\mu_1 h_1}{2R_1}}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\sigma = \sqrt{\frac{2\mu}{a}}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$T = \sqrt{\frac{2\mu}{a}}$</td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>Viscoelastic $0 &lt; e &lt; 1$</td>
<td>$F = \frac{4}{3} R_1^{0.5} E_1^e \delta^{1.5}(1 + 1.5\alpha\delta)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$e = 1 - \alpha v_2$</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>Viscoelastic $0 &lt; e &lt; 1$</td>
<td>$F(t) = k_0 \delta^{1.5} + k_1 x^{4.5} \lambda$</td>
<td></td>
</tr>
</tbody>
</table>


Phase III is that the deformed fruit rebounds upward to a height $h_2$ because the elastic deformation energy $\Delta E_e$ is recovered, which involves the hypothesis that the viscous response of the fruit has an insignificant influence on the recovery of elastic deformation energy. In the meanwhile, with the action of gravity acceleration $-g$, the velocity changes from $v_2$ to 0 because the kinetic energy after impact is fully transformed into potential energy $mgh_2$.

Multiple free-fall drops will increase the plastic deformation energy of fruit, causing more serious mechanical damage. During storage, the initial damage will usually lead to subsequent accelerated rot of a whole fruit. Therefore, after postharvest handling, a tiny aggregation of damaged cells can eventually cause the fruit to be discarded in the market. After the initial free drop and rebound, the fruit will similarly free drop from $h_3$. The difference between $I$ and $IV$ is the drop height. During free-fall drop tests, force–time data and acceleration–time data at Phase II can be recorded and then the extracted peak force $F_{\text{max}}$ and impact duration $T$ are used to characterize the impact behavior. The trends of velocity–time and acceleration–time in a free-drop cycle are shown in Figure 4. Some related impact models of free-fall drop are listed in Table 4.

Impact models of free-fall drop. In model 18 (Table 4), Metz (2006) presented a basic method for the calculation of impact mechanical parameters such as maximum acceleration $\alpha_{\text{max}}$ and peak impact force $F_{\text{max}}$ when an object is free-fall-dropped onto a force sensor. The assumption was that the object will have a perfect rebound after drop impact, which approximates a steel-on-steel impact. The initial and final velocities are equal in magnitude but opposite in direction (Metz 2006). Model 18 can be used to roughly assess the drop impact mechanics of unripe fruits because unripe fruits are relatively firm.

Hunter (1957) proposed a half-sine model to predict the contact force, impact duration, and elastic deformation of a solid object during free-fall drop impact (model 19, Table 4). It was deduced...
using Hertzian nonadhesive elastic contact theory and some hypotheses that: (i) the deformation of the object generated by the impact is elastic and concentrated in small contact regions; (ii) the velocity \( v_f \) is below 10 m/s so that air resistance can be considered negligible; and (iii) the acceleration–time data of the object during impact follows a half-sine function (Hunter 1957; Zhou and others 2008; Tempelman and others 2012). The peak acceleration \( \frac{\pi}{2} \left( \frac{K_u + K_p}{r} \right) \) deduced by the half-sine model is slightly smaller than the value from model 18. The 1st hypothesis can be met in some free-fall drop tests on fruits and vegetables. It has been shown that the impact deformation will be elastic when the falling height of Hongfushi apple, Plum tomato, and Kent mango is lower than 4 cm (Lu 2009), 1.5 cm (Lien and others 2009), and 10 cm (Hahn 2004), respectively. The threshold height for drop damage can also be evaluated by a nonlinear equation including 5 variables (fruit radius, shear strength, elastic modulus, mass, and acceleration due to gravity) (Horsfield and others 1972). Therefore, this will have the potential to be a nondestructive inverse method for predicting fruit mechanics and texture.

In model 20 (Table 4), Johnson (1987) proposed an equation to predict the yield strength of elastic–plastic spheres at low-speed drop impacts. In model 21, developed by Horsfield and others (1972), it was assumed that (i) the fruit material is homogeneous and (ii) the fruit flesh fails in shear rather than in tension. The maximum internal shear stress is equal to 0.27 times the maximum compressive stress at the center of the contact area after a drop impact. The relationship between the ratio of maximum shear stress to shear strength and the number of drop impacts follows log \( I_{\text{max}}/S_0 = -0.069 \log N \). The critical height for drop damage of a peach fruit dropped onto another fruit or a rigid surface can be assessed accurately by the 2 general equations in model 21 if the shear strength of fruit material is given. This model is based on experiments on peaches by considering fruit drops against various elements of harvesting and handling systems. It provides criteria for the design of such equipment that will minimize bruising. The modeling method can be extended to other fruits, but the model itself cannot be directly used for other fruits.

Kabas and others (2008) proposed a finite element model to simulate free-fall drop tests of cherry tomatoes in order to investigate the relationship between drop height and critical damage (model 22, Table 4). The threshold height for damage evaluated by model 21 was set as the drop height in the finite element simulation, and then the drop impact was simulated to solve the stress distribution in the internal tissues of the fruit. The maximum Von Mises stress corresponds to the critical damage stress. A similar method was also used for peach fruits (Kabas and Vladaut 2015). This FEM can also be used to solve the critical drop height of fruit with known shear failure strength. Celik and others (2011) predicted the internal maximum stress and contact force of a free fall drop of Golden Delicious apple by a finite element model, and they compared the simulated impact and rebound processes of fruits with the experiment recorded by high-speed camera. The FEM can be used to explore further the relationship between force and deformation of fruits during drop impact and rebound and to validate whether the contact theory of Hertz can be used in impact analysis, because it is difficult to measure force–deformation data of fruit during impact using existing equipment. Emanuele Cerruto and others (2015) developed a finite element model of potato tuber to evaluate the effect of drop height, size, density, elastic modulus on the maximum impact force, and internal central acceleration during drop impact. Subsequently, a miniaturized acceleration measuring unit AMU was implanted into the center of a real potato tuber to determine the acceleration of the center during drop impact tests (Geyer and others 2009). The simulations provided acceleration values about twice as large as those measured in the experiments with the AMU device. This is an interesting attempt to explore the real internal mechanical response of vegetables during drop impact. There will be a potential challenge in investigating the force transmission mechanism between fruit tissues or cells but this would be highly valuable.

Meggitt (2009) proposed 3 simple equations in model 23 (Table 4) to predict the perfectly plastic impact behavior of a deformable solid object in free-fall drops onto a rigid surface. It was assumed that the relationship between impact force and deformation is elastic–perfectly plastic, the contact area between object and rigid surface during impact is constant and there is no rebound after impact (MEGGITT 2009). A greater acceleration level is achieved with a larger surface area, a higher drop, a lower mass fruit or a stiffer fruit (higher modulus), and the peak impact force depends on the combined action of the fruit mass and the acceleration during impact (MEGGITT 2009). This model is very suitable for predicting the drop impact phenomena of rectangular tissue blocks of fruits and inversely to evaluate the mechanics and texture of fruit tissues.

In model 24 (Table 4), Hunt and Crossley (1975) proposed that the impact force of a viscoelastic sphere onto a rigid surface was made up of a Hertz elastic term and a damping term. The magnitude of the damping term is proportional to the deformation to the power of 1.5. In model 25 (Table 4), Lu (2009) proposed that the magnitude of the damping term is proportional to the deformation of Fuji apples to the power of 4.5. In model 25, dropped apples were assumed to be viscoelastic, which is much closer to their natural properties. The modeling method can also be used to fit the force (or acceleration) deformation (or velocity) data of other fruits and to evaluate the impact energy and absorbed energy during drop impact.

**Future Research Directions**

Likely directions for future research on compression and impact mechanics, related to postharvest fruits, can be summarized as follows:

1. **Prediction of compression and impact mechanics of unripe and half-ripe fruit materials, especially internal tissues/microscopic cuticle and single cells in the unharvested fruit.** Most of current and past research focused on ripe fruit materials from markets. However, most fruits are harvested at the unripe stage and then mechanically handled and transported to markets in an increasingly ripening state. One of the important aims in scientific research is to discover how to avoid mechanical damage (such as yield and rupture) of unripe fruits during harvest and postharvest handling (Prasanna and others 2007). Therefore, it is essential to develop accurate mathematical models to relate the mechanical properties of fresh fruits to their different ripening stages for predicting the mechanics of fresh fruit materials (including macroscale organs, mesoscale tissues, and microscale cells) before ripeness.

2. **Comparison of the predictive performance of different mechanical models for fruit materials.** According to Table 1 to 4, the compression or impact mechanical properties of a fruit material can be predicted from different mechanical models. It is unclear which model might be more accurate/suitable to predict fruit mechanics. Therefore, it is
necessary to validate and compare the predictive performance of different models to real fruit mechanics. Furthermore, it would be valuable to investigate the influence of the real shapes of fruits or single cells on mechanical parameter values predicted from any given mechanical model.

(3) Studies on multiple/repetitive compression and impact mechanics of fruit materials. Fresh fruits are viscoelastic-plastic, and during mechanical handling, there are always multiple compressions or impacts by other fruits and rigid surfaces at the same/different locations of the fruit surface. Although there are limited models on multiple loading of elastic-plastic spheres, research on the mechanical behavior, especially plastic, of fruit/tissue/cell materials in multiple/repetitive compression/impact motions is very scarce. It would be valuable to explore deeply the multiple elastic-plastic compression/impact behavior of fruit materials from the microscale to macromolecular level, with the guided assistance of models.

(4) Improvements in characterization of the failure behavior of fruit materials. Fruit is a living multiscale hierarchical structural material and its failure behavior is different from nonliving materials such as metals. However, existing failure criteria are always validated as being effective based on the failure characteristics of nonliving materials. The resulting failure phenomena do not fully follow the failure criteria of fruits used in FEM/DEM simulations in the previous study. Therefore, it is important to characterize the microscopic failure behavior of fruit materials, especially internal tissue/cell damage and microcracks; and then to develop suitable failure criteria for such materials.

(5) FEM/DEM modeling and simulation in the investigation of compression and impact mechanics of fruit materials could be enhanced, especially impact mechanics. Some real situations should not be neglected in modeling and simulation, such as irregular geometry and asymmetric internal structure, multiscalar components and multiple cell types in a fruit, viscoelastic-plastic material properties, magnitude of water loss of fruit/tissue/cell during loading, contact model selection, and real postharvest working conditions. It is necessary to develop real fruit and cell models to simulate the relationship between physical–geometrical properties and compression/impact mechanics and then to compare the results from model prediction and experimental analysis. The application of the simulating models to real fruit behavior in real postharvest mechanical handling conditions is crucial to relate those models to real fruit damage.

(6) Enhancement in interaction analysis, both theoretical and experimental. Theoretical analysis of the mechanics from compression and impact models between 2 spheres and between a sphere and a rigid plate has generated many excellent conclusions. However, most of the conclusions, especially from models 4, 5, 11, 12, 14, 19, and 20, have rarely been used in the investigation of mechanical behavior of fresh fruits during compression and impact tests. Furthermore, it is necessary to make some comparisons of the predictions from selected models for validation of sensitivity and accuracy. There are some theoretical models on the yield behavior of spherical materials, but only little information on the macroscopic deformation and rupture of elastic-plastic spheres; so, the extension of theoretical models to, and the experimental exploration of, spherical fruits is necessary.

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Nomenclature

- $a, a(t)$: impact acceleration
- $a_{vg}$: mean impact acceleration
- $a_i$: cleavage depth of indentation
- $a_d$: dimensionless yielding inception depth
- $a_{max}$: maximum impact acceleration
- $d$: thickness of cell wall
- $e$: coefficient of restitution
- $g$: gravity acceleration
- $h_1$: drop height
- $h_2$: rebound height
- $h_{1}'$: critical drop damage height
- $h_{2}'$: critical rebound damage height
- $m^*$: equivalent mass
- $r$: relative radius of contact circle
- $r_i$: major radius of contact ellipse
- $r_2$: minor radius of contact ellipse
- $t_i$: compression duration
- $E_i$: apparent elastic modulus in compression
- $E_c$: elastic modulus of cell wall
- $E_{el}$: instantaneous elastic modulus
- $E_{el i}$: elastic modulus of $i$th spring
- $F(t)$: instantaneous contact force
- $F_i$: dimensionless force
- $F_i(R'/R'')$: compression force or impact force
- $F_{max}$: peak impact force
- $F_{2}$: impact/compression force at yield point
- $G_0$: instantaneous elastic modulus in relaxation
- $G$: equilibrium elastic modulus in relaxation
- $K_{IC}$: critical stress intensity factor
- $K_{ni}$: viscous coefficient of $i$th dashpot
- $N$: number of impacts
- $R_i, m_i, E_i$: generalized radius, mass, and elastic modulus
- $R_i, R_i'$, radius, mass, Poisson’ ratio, and elastic modulus of the body $i$ ($i = 1$ and 2), respectively.
- $R_e$: equivalent radius of curvature
- $S$: contact area
- $U$: impact/collision duration
- $V^*$: characteristic velocity
- $V_i$: relative velocity at impact yield
- $V_i'$: relative velocity of approach of 2 bodies
- $V_{si}$: relative velocity of 2 bodies after impact
- $X, \delta^*$: maximum deformation of contact body
- $\Delta E$: loss of kinetic energy
- $\Delta E_{el}$: elastic deformation energy
- $\Delta E_{pl}$: plastic deformation energy
- $\delta, \delta(t)$: instantaneous deformation
- $\delta_y$: deformation at yield
- $\delta_{yj}$: peak displacement
- $\delta_i$: elastic limit of deformation
- $\epsilon$: fractional deformation
- $\dot{\epsilon}$: dimensionless fractional deformation

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\[ \lambda \] 
stretch ratio

\[ v_i, v_i' \]
velocity of the body \( i (i = 1 \) and \( 2) \) before and after collision, respectively.

\[ \xi, \eta \]
two coefficients of viscosity of the body which are related to \( k_c \).

\[ \rho, A \]
coefficients of elastic and viscous, respectively.

\[ \sigma_{\text{max}} \] 
maximum compression stress

\[ \sigma_y \] 
yield strength

\[ \sigma_f \] 
fracture strength

\[ \sigma_1, \sigma_3 \]
maximum and minimum principal stresses, respectively.

\[ \tau_{\text{max}} \]
maximum and minimum shear stresses, respectively.

\[ \tau_{i,i} \]
relaxation time of \( i\)th element

\[ \tau_{\text{bo}} \]
bruising strength

\[ \tau_y \]
shear strength of the softer body

References


Mechanical models of compression and impact...


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